Technical Notes

Vibrational Characteristics of Thin-Wall Conical Frustum Shells

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Nomenclature

E = Young's modulus of elasticity
f_N = natural frequency, cps
g = gravitational constant
h = shell thickness
l = length of cone measured normal to the base
m = longitudinal mode; number of circumferential nodes for free-free models, one more than the number of circumferential nodes between the ends of the model for fixed-free models
n = circumferential mode number or number of circumferential waves
r_0 = minor radius of conical frustum
r_1 = major radius of conical frustum
α = semivertex angle of conical frustum
γ = weight density of shell material
λ = l/r_0
ν = Poisson's ratio
ρ = mass density of shell material, $\rho = \gamma/g$
Δ = dimensionless frequency parameter

Subscripts

i = inextensional
e = extensional

I. Introduction

THIN-WALL conical frustum shells are relevant to such components of space flight structures as interstage adapters and engine nozzles. As the tendency toward thinner and consequently more flexible components continues, greater emphasis must be placed on the reaction of such structures to applied forces. Although considerable attention has been devoted to the static and dynamic behavior of thin-wall cylinders having various boundary conditions, little information pertinent to the properties of conical frustum shells has been published.[1] Yet, the characteristic vibrational properties of the conical frustum are essential to the determination of their response to dynamic loads. The purpose of this paper is to present experimental data for conical frustums having free-free and fixed-free boundary conditions and to compare the fixed-free experimental data with analytical results obtained for this case by use of the Rayleigh-type vibration analysis by Platus.[2,3]

II. Experimental Apparatus and Procedure

Structural models

Each frustum consisted of three identical sections of 0.007-in.-thick stainless steel with overlapped, spot-welded longitudinal seams. Pertinent dimensions are given in Table 1. Ratios of thickness to average radius varied from 0.000538 to 0.000700, values that approximate those of current launch vehicles.

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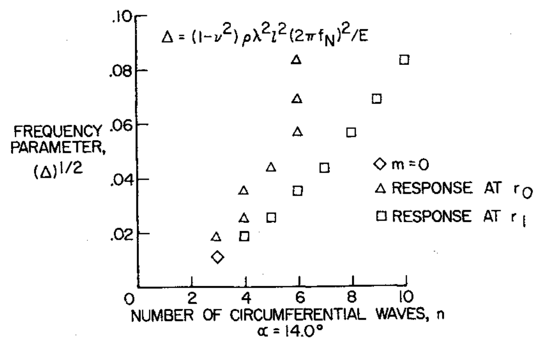


Fig. 1 Experimental natural frequency characteristics of a circular conical frustum with free-free end conditions.

Instrumentation and test procedure

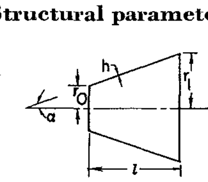
Natural frequencies of the structures were excited by small electromagnetic shakers and measured with an electronic digital counter. One of two transducers, either a crystal accelerometer or a velocity probe, was employed to determine the response of the structure to the induced vibration. The amplified transducer output was applied to the vertical deflection plates of a cathode ray oscilloscope while the oscillator output signal was applied to the horizontal plates, resulting in a Lissajous figure on the scope. Natural frequencies and nodal points were detected by observing shell displacement amplitudes and Lissajous figures. The mass added by the electromagnetic shakers was found to have a negligible effect upon natural frequencies and mode shapes since equivalent results were obtained by use of an air shaker.

Each model was tested with free-free and fixed-free boundary conditions. To simulate free-free conditions, the models were suspended by six strings spaced evenly about the circumference of the shell. For fixed-free support, frustums 1, 2, and 3 were rigidly clamped at the minor diameter by two plates and dogged to a massive backstop. It was found necessary to imbed frustum 4 in a metal alloy in order to approach the requirement of zero slope at the fixed end.

III. Discussion of Results

The experimental results for free-free frustum 3, typical of the results of all the free-free frustums, are given in Fig. 1. It was found that at higher natural frequencies there are a greater number of circumferential waves at the major diameter than at the minor diameter. The difference in the number of waves increased with conicity, ranging in difference from one to five for the frequency range covered. A typical nodal pattern for a difference of two waves is shown in Fig. 2.

Table 1 Structural parameters of models



STRUCTURE	α, DEG	h, IN.	l, IN.	r_0, IN.	r_1, IN.
FRUSTUM 1	3.2	0.007	36	12	14
FRUSTUM 2	7.4	0.007	30	10	14
FRUSTUM 3	14.0	0.007	24	8	14
FRUSTUM 4	24.0	0.007	18	6	14

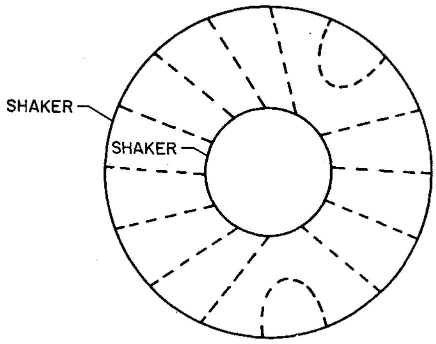


Fig. 2 Sketch of typical nodal pattern for free-free frustums as viewed along the longitudinal axis.

Thus far, no adequate analysis for prediction of the natural frequencies of frustums of the type under investigation with free-free boundary conditions is known to the authors. An analysis[4] has been developed, however, which predicts a difference in the number of circumferential waves for radially stiffened frustums fixed (imbedded) at the major diameter.

References 2 and 3 develop frequency equations for fixed-free frustums. Two frequency equations are derived, one for inextensional (i.e., bending or flexure) and another for extensional or membrane vibrations. Shell displacements are assumed for each case and substituted into the potential and kinetic energy expressions that are then set equal. The extensional analysis results in a rather complicated matrix that can be solved for the dimensionless frequency parameter Δ_e, which is related to the extensional frequencies as follows:

$$\Delta_e = (1 - \nu^2)\rho\lambda^2 l^2 (2\pi f_e)^2/E \tag{1}$$

Inextensional frequencies were calculated from the equation

$$f_i = \frac{hn(n^2 - 1)}{4\pi r_0^2 \cos\alpha}\left[\frac{E}{3\rho(1 - \nu^2)}\right]^{1/2}\left(\frac{N}{D}\right)^{1/2} \tag{2}$$

where N and D are dimensionless geometric parameters.[2, 3] The frequency parameter Δ_i is then obtained by replacing f_e in Eq. (1) with f_i obtained from Eq. (2).

The theoretical frequency parameter was then formed from the relation

$$(\Delta)^{1/2} = (\Delta_i + \Delta_e)^{1/2} \tag{3}$$

A comparison of the experimental and theoretical value of the frequency parameter for the fixed-free frustums is shown in Fig. 3.

IV. Concluding Remarks

Results of Rayleigh-type vibration analyses[2, 3] have shown good agreement with experimental results for conical frustum shells with fixed-free ends. No adequate theory is known to the authors for predicting the mode shapes and natural frequencies of free-free frustums. An essential ingredient of desired analysis is its generality to take into account the difference in the number of circumferential waves found at the major and minor diameters of the shell. In addition, a need for better analytical procedures exists for the determination of the dynamic behavior of various cylinder-cone combinations subjected to vibratory loads under all boundary conditions.

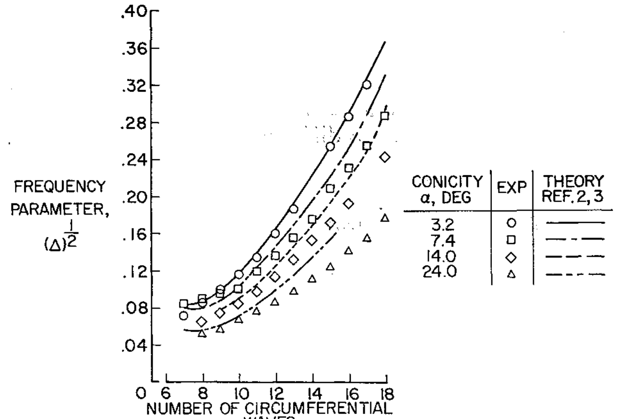


Fig. 3 Experimental and theoretical frequency characteristics of conical frustum shells with fixed-free end conditions, $m = 1$.

References

[1] Gros, C. G. and Forsberg, K., "Vibrations of thin shells: A partially annotated bibliography," Lockheed Special Bibliography SB-63-43 (April 1963).

[2] Platus, D. H., "Study on bell-mode vibrations of conical nozzles, first quarterly progress report," Aerojet-General Rept. 0660-1, 3-14 (August 1962).

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[4] McGrattan, R. J. and North, E. L., "Shell mode coupling," General Dynamics, Electric Boat Rept. U411-63-005 (March 1963).

Use of Stress Relaxation Tests to Characterize Time Dependencies of a Composite Solid Propellant

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Introduction

To define or characterize the behavioral characteristics of a viscoelastic material accurately, its responses to an applied load must be determined as a function of strain rate, time, and temperature. These characteristics may be determined by means of a stress relaxation test. This test requires an instantaneous specimen extension and the measurement of the diminishing force required to maintain the elongation constant over an extended time period.

Experimental Procedure

The samples used in the stress relaxation determinations were cast cylindrical specimens, designed for a constant gage length, with a 0.5-in. diam in the necked-down region.[1] The total specimen length was 3.25 in., with a necked-down length of 1.8 in. Aluminum end tabs were bonded to the specimen with Shell 911-S adhesive. The specimens were attached to the statically mounted load cell and the upper, or movable jaw.

Extremely rapid specimen extension was achieved by actuating a gas-operated piston by nitrogen pressure. Both sides of the piston were pressurized. The pressure above the piston was then released rapidly by the actuation of a solenoid valve. The movement of the upper jaw of the apparatus was prevented until the pressure differential was sufficient to break a shear pin and extend the specimen in tension at a rate of 10,500 in./min.

A positive locking cone was machined in the upper plate, with a matching cone on the upper jaw to maintain the required specimen alignment during elongation and to prevent bouncing of the jaw at the end of the stroke. This cone also creates an air cushion that acts as a retarding force at the end of the stroke.

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